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Research Article

Vibration Reduction for a Flexible Arm Using Magnetorheological Elastomer Vibration Absorber

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The application of the magnetorheological elastomer (MRE) to nonlinear vibration control for a flexible arm is investigated in this paper. A semiactive control method is suggested to reduce vibration via the internal resonance and the MRE. To establish a vibration energy transfer channel, a tuned vibration absorber based on the MRE is developed. Through adjusting the coil current, the frequency of the vibration absorber can be readily controlled by the external magnetic field, thereby maintaining the internal resonance condition with the flexible arm. By the perturbation analysis, it is proven that the internal resonance can be successfully established between the flexible arm and the MRE vibration absorber, and the vibration energy of the flexible arm can be transferred to and dissipated by the MRE vibration absorber. Through numerical simulations, virtual prototyping simulations, and experimental investigation, it is verified that the proposed method and the suggested MRE vibration absorber are effective in controlling nonlinear vibration of the flexible arm.

1. Introduction

Vibration absorption is a type of effective methods for attenuating strong vibration of the mechanical systems. Various dynamic vibration absorbers (DVAs) are widely used to reduce the forced vibration excited by external harmonic excitations with specific frequency.

Since passive DVAs possess considerably narrow frequency bandwidth, they lack enough adaptability. Therefore, a number of measures for tuning the frequencies of DVAs have been developed, including tuning the curvature of two parallel curved beams [1], changing effective coil number of a spring [2], controlling the space between two spring leaves [3], adjusting the length of threaded flexible rods [4], changing effective length of a flexible cantilever beam by moving the intermediate support [5], varying the pressure of air springs [6], and adopting a variable magnetic spring controlled by current [7]. Although they are able to successfully adjust the frequencies, most of them face such challenges as large dimensions, large weight, slow adjusting speed, and high energy consumption. Recently, some emerging smart materials have potential to deal with these problems, inducing shape memory alloy [8], magnetorheological elastomers (MRE) [9], and piezoelectric ceramic [10].

The MRE is one of the most popular smart materials, whose modulus can be rapidly, continuously, and reversibly tuned by adjusting the external magnetic field. A number of studies have put forward various tuned vibration absorbers based on the MRE [11–13] and obtained satisfactory vibration control results. Nevertheless, most of them are only effective in linear vibration problem of the primary system. For a flexible arm characterized by distributed flexibility and rigid motion, nonlinear terms cannot be ignored and thus many control methods based on linear vibration model will be invalid. In addition, these methods have to rely on specific information of external excitations, for example, the position and frequency, to neutralize vibration of the primary system. As a result, if the external excitations are unknown or unpredictable in such case as the outer space, they will deteriorate. According to the above analyses, it is worth investigating an effective vibration absorption method based on the MRE vibration absorber to reduce nonlinear vibration excited by external excitations.

Internal resonance is a typical modal interaction phenomenon of the multi-degree-of-freedom nonlinear dynamics system. If one mode's frequency is commensurable or nearly commensurable with another mode's frequency, the internal resonance probably can be established between them. Via the internal resonance, an internal channel for transferring vibration energy can be built between these two modes. If one mode's vibration is excited, its energy can be transferred to the other mode by way of the internal resonance, and vice versa. Golnaraghi et al. [14, 15] firstly proposed an idea to reduce vibration of flexible cantilever beam using the internal resonance. Afterwards, Tuer et al. [16], Duquette et al. [17], and Oueini and Golnaraghi [18] further performed related theoretical and experimental study. However, their flexible cantilever beam model is actually a rigid beam connected by a linear torsional spring. Obviously, this model is not suitable for the analysis of real flexible arm with distributed flexibility. In recent years, some research has been conducted on the flexible arm. Pai et al. [19] used higher order internal resonance to design a vibration absorber to reduce vibration of a cantilevered plate. Yaman and Sen [20] absorbed vibration of a cantilever beam using a pendulum. Hui et al. [21] attenuated translational vibration of the source mass by transferring the resonant energy from the symmetrical to antisymmetrical mode. However, their studies only concern flexible structure without rigid motion and thus cannot be used to solve much more complex vibration problem of the flexible arm undergoing large-scale joint motion. Furthermore, the primary system in these studies is assumed as a linear vibration model for the simplicity, while the flexible arm itself is a complex nonlinear dynamics system. As a result, arbitrary linearization will result in fundamental mistakes. In our previous work, a virtual vibration absorber is put forward to reduce nonlinear vibration of the flexible arm but is restricted to the output power of the servomotor [22]. To the best of our knowledge, there are few theoretical and experimental works about semiactive vibration control for the flexible arm based on both the internal resonance and the MRE vibration absorber. Its theoretical correctness and practical feasibility need to be studied and verified.

The aim of the paper is to research the application of the MRE to nonlinear vibration control for the flexible arm characterized by distributed flexibility and rigid motion via the internal resonance. To this end, a semiactive control method is put forward to attenuate nonlinear vibration of a flexible arm based on both the internal resonance and the MRE. This paper consists of eight sections. Following the introduction, the dynamics model of a flexible arm and a MRE vibration absorber is built in Section 2. Section 3 performs perturbation analysis on the dynamics equations concerning the fundamental mode of the flexible arm and the mode of the vibration absorber. In Section 4, whether the internal resonance can be successfully established is studied and vibration reduction based on the internal resonance is proven. Afterwards, a series of numerical simulations and virtual prototyping simulations are performed in Section 5 to verify the correctness of the theoretical analysis. Next, a shear mode vibration absorber with the MRE is developed in Section 6 and several experimental studies are conducted in Section 7. In the final section, the conclusion is summarized.

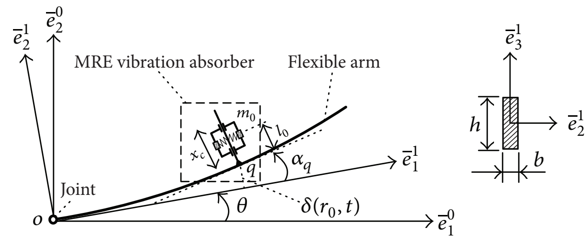

Figure 1: Dynamics model of the system.

2. Dynamics Model

In this study, the simplified dynamics model of the flexible arm and the proposed MRE vibration absorber (whose detailed configuration is exhibited in Section 6) is shown in Figure 1. The flexible arm is viewed as a uniform Euler-Bernoulli beam with the length l, the rectangle cross-section of height h, and width b. It is linked to a rigid joint driven by a motor, whose nominal motion is denoted by θ. Only transverse deformation $\delta(x_1, t)$ about $\bar{e}_2^1$ axis is considered, where t is the time and x_1 is the distance measured from o along the $\bar{e}_1^1$ axis. The MRE vibration absorber is simplified as a slider mass-spring-dashpot device with the mass m_0, stiffness k, damping c, equilibrium position l_0, and reciprocating motion displacement x_c. It is attached to the flexible arm at the point q (i.e., $x_1 = r_0$). The angle of the tangent of the point q with respect to $\bar{e}_1^1$ axis is denoted by α_q.

The transverse deformation of an arbitrary point p in the flexible arm can be expressed as

$$\bar{\delta}_p^1 = \left[\sum_{k=1}^{n} \delta_{pk}^1 \varphi_k(t) \right] \bar{e}_2^1, \tag{1}$$

where $\varphi_k(t)$ is the kth modal coordinate describing deformation of the flexible arm, δ_{pk}^1 is the kth mode shape satisfying certain geometric and force boundary conditions, and n is the number of mode shapes.

Since most of the vibration energy is often converged in the low order modes, only the fundamental mode is considered in this study due to its dominant contribution to the vibration response. As a result, (1) can be written as

$$\bar{\delta}_p^1 \doteq \delta_{p1}^1 \varphi_1(t) \bar{e}_2^1. \tag{2}$$

Similarly, the transverse deformation of the point q (i.e., the position of the vibration absorber) can be written as

$$\bar{\delta}_q^1 \doteq \delta_{q1}^1 \varphi_1(t) \bar{e}_2^1. \tag{3}$$

For purpose of reducing vibration via the vibration absorber, the dynamics effects between the fundamental mode coordinate φ_1 of the flexible arm and the reciprocating motion x_c of the vibration absorber need be analyzed. To this end, the dynamics equations concerning φ_1 and x_c are derived using Kane's method and written as

$$\begin{aligned}
&-m_0\ddot{\theta}\delta_q^1 r_0 + m_0\dot{\theta}^2\delta_q^1 l_0 - m_0\dot{\theta}^2 l_0 r_0\frac{d\delta_q^1}{dx} - m_0\ddot{\theta}{l_0}^2\frac{d\delta_q^1}{dx} \\
&\quad - \int_0^l \rho\ddot{\theta}\delta_q^1 x\,dx + 2m_0\delta_q^1 l_0\left(\frac{d\delta_q^1}{dx}\right)^2\varphi_1(t)\,\ddot{\varphi}_1(t) \\
&\quad - m_0\left(\delta_q^1\right)^2\ddot{\varphi}_1(t) - m_0{l_0}^2\left(\frac{d\delta_q^1}{dx}\right)^2\ddot{\varphi}_1(t) \\
&\quad - 2m_0 l_0\left(\frac{d\delta_q^1}{dx}\right)^2 x_c\ddot{\varphi}_1(t) - \int_0^l \rho\left(\delta_q^1\right)^2 dx\,\ddot{\varphi}_1(t) \\
&\quad + m_0\delta_q^1 l_0\left(\frac{d\delta_q^1}{dx}\right)^2 {\dot{\varphi}_1}^2(t) \\
&\quad - 2m_0 l_0\left(\frac{d\delta_q^1}{dx}\right)^2 \dot{x}_c\dot{\varphi}_1(t) \qquad (4)\\
&\quad + m_0\ddot{\theta}l_0 r_0\left(\frac{d\delta_q^1}{dx}\right)^2\varphi_1(t) - \int_0^l \mathrm{EI}\delta_p^1\frac{d^4\delta_p^1}{dx^4}dx\,\varphi_1(t) \\
&\quad - m_0\delta_q^1\ddot{x}_c - 2m_0\dot{\theta}l_0\frac{d\delta_q^1}{dx}\dot{x}_c - 2m_0\ddot{\theta}l_0\frac{d\delta_q^1}{dx}x_c = 0, \\
&kx_c + c\dot{x}_c + m_0\ddot{x}_c - m_0 l_0\left(\frac{d\delta_q^1}{dx}\right)^2 {\dot{\varphi}_1}^2(t) \\
&\quad - 2m_0\dot{\theta}l_0\frac{d\delta_q^1}{dx}\dot{\varphi}_1(t) + m_0\delta_q^1\ddot{\varphi}_1(t) + m_0\ddot{\theta}r_0 \\
&\quad - m_0\dot{\theta}^2 l_0 = 0,
\end{aligned}$$

where ρ is mass per length of the flexible arm and EI is the flexural rigidity.

3. Nonlinear Solution

3.1. Nondimensionalization. To solve (4), they must be nondimensionalized in advance. Therefore, the nondimensional variables φ^* and x_c^* are defined by

$$\begin{aligned}
\varphi^* &= \frac{\varphi_1}{l}, \\
x_c^* &= \frac{x_c}{l}.
\end{aligned} \qquad (5)$$

For convenience, φ^* and x_c^* are still, respectively, described by φ and x_c after the nondimensionalization. The dynamics equations are transferred as follows:

$$\begin{aligned}
&\ddot{\varphi}_1 + 2\xi_q\omega_1\dot{\varphi}_1 + {\omega_\varphi}^2\varphi_1 \\
&\quad = g_1 + g_2 + g_3\varphi_1\ddot{\varphi}_1 - g_4 x_c\ddot{\varphi}_1 + g_5{\dot{\varphi}_1}^2 - g_4\dot{x}_c\dot{\varphi}_1 \\
&\qquad + g_6\varphi_1 - g_7\ddot{x}_c - g_8 x_c, \qquad (6)\\
&\ddot{x}_c + 2\xi_c\omega_2\dot{x}_c + {\omega_c}^2 x_c = h_1{\dot{\varphi}_1}^2 - h_2\ddot{\varphi}_1 - h_3 + h_4,
\end{aligned}$$

where

$$\begin{aligned}
g_1 &= \frac{-\left(m_0\delta_q^1 r_0 + m_0{l_0}^2\left(d\delta_q^1/dx\right) + \int_0^l \rho\delta_p^1 x\,dx\right)\ddot{\theta}}{l\left(m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx\right)}, \\
g_2 &= \frac{\left(m_0\delta_q^1 l_0 - m_0 l_0 r_0\left(d\delta_q^1/dx\right)\right)\dot{\theta}^2}{l\left(m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx\right)}, \\
g_3 &= \frac{2l^2 m_0\delta_q^1 l_0\left(d\delta_q^1/dx\right)^2}{m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
g_4 &= \frac{2ll_0 m_0\left(d\delta_q^1/dx\right)^2}{m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
g_5 &= \frac{ll_0 m_0\delta_q^1\left(d\delta_q^1/dx\right)^2}{m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
g_6 &= \frac{l_0 m_0 r_0\left(d\delta_q^1/dx\right)^2\ddot{\theta}}{m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
g_7 &= \frac{m_0\delta_q^1}{m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \qquad (7)\\
g_8 &= \frac{2m_0 l_0\ddot{\theta}\left(d\delta_q^1/dx\right)}{m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
h_1 &= ll_0\left(\frac{d\delta_q^1}{dx}\right)^2, \\
h_2 &= \delta_q^1, \\
h_3 &= \frac{r_0}{l}\ddot{\theta}, \\
h_4 &= \frac{l_0}{l}\dot{\theta}^2, \\
\omega_\varphi^2 &= \frac{\int_0^l \mathrm{EI}\delta_q^1\left(d^4\delta_q^1/dx^4\right)dx}{m_0\left(\delta_q^1\right)^2 + m_0{l_0}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
\omega_c^2 &= \frac{k}{m_0}, \\
\xi_c &= \frac{c}{2\sqrt{m_0 k}}.
\end{aligned}$$

On the other hand, to make the nonlinearities appear in the same perturbation equations, let $\omega_c/\omega_\varphi = \omega_s$, $t = \omega_1\tau$, $\varphi_1 = \varepsilon\widehat{\varphi}_1$, $x_c = \varepsilon\widehat{x}_c$, $\xi_q = \varepsilon\widehat{\xi}_q$, and $\xi_c = \varepsilon\widehat{\xi}_c$, where ε is a small nondimensional bookkeeping parameter, $0 < \varepsilon \ll 1$. For convenience, $\widehat{\varphi}_1$, $\widehat{x}_c$, $\widehat{\xi}_q$, and $\widehat{\xi}_c$ are still, respectively, described by φ_1, x_c, ξ_q, and ξ_c after the nondimensionalization. Then, (6) can be expressed as

$$\begin{aligned}
&\ddot{\varphi}_1 + 2\varepsilon\xi_q\dot{\varphi}_1 + \varphi_1 \\
&\quad = e_1\ddot{\theta} + e_2\varepsilon\dot{\theta}^2 + e_3\varepsilon\varphi_1\ddot{\varphi}_1 - e_4\varepsilon x_c\ddot{\varphi}_1 + e_5\varepsilon\dot{\varphi}_1{}^2 \\
&\qquad - e_6\varepsilon\dot{x}_c\dot{\varphi}_1 + e_7\varepsilon\ddot{\theta}\varphi_1 - e_8\ddot{x}_c - e_9\varepsilon\ddot{\theta}\varphi_1, \\
&\ddot{x}_c + 2\varepsilon\xi_c\omega_s\dot{x}_c + \omega_s{}^2 x_c \\
&\quad = h_1\varepsilon\dot{\varphi}_1{}^2 - h_2\ddot{\varphi}_1 - h_3\ddot{\theta} + h_4\varepsilon\dot{\varphi}_1{}^2,
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
e_1 &= \omega_1{}^2, \\
e_2 &= \frac{-\left(m_0\delta_q^1 r_0 + m_0 l_0{}^2\left(d\delta_q^1/dx\right) + \int_0^l \rho\delta_p^1 x\,dx\right)\ddot{\theta}}{l\left(m_0\left(\delta_q^1\right)^2 + m_0 l_0{}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx\right)}, \\
e_3 &= \frac{\left(m_0\delta_q^1 l_0 - m_0 l_0 r_0\left(d\delta_q^1/dx\right)\right)\dot{\theta}^2}{l\left(m_0\left(\delta_q^1\right)^2 + m_0 l_0{}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx\right)}, \\
e_4 &= \frac{2l^2 m_0\delta_q^1 l_0\left(d\delta_q^1/dx\right)^2}{m_0\left(\delta_q^1\right)^2 + m_0 l_0{}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
e_5 &= \frac{2ll_0 m_0\left(d\delta_q^1/dx\right)^2}{m_0\left(\delta_q^1\right)^2 + m_0 l_0{}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
e_6 &= \frac{ll_0 m_0\delta_q^1\left(d\delta_q^1/dx\right)^2}{m_0\left(\delta_q^1\right)^2 + m_0 l_0{}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
e_7 &= e_5, \\
e_8 &= \frac{l_0 m_0 r_0\left(d\delta_q^1/dx\right)^2\ddot{\theta}}{m_0\left(\delta_q^1\right)^2 + m_0 l_0{}^2\left(d\delta_q^1/dx\right)^2 + \int_0^l \rho\left(\delta_p^1\right)^2 dx}, \\
e_9 &= e_2.
\end{aligned} \tag{9}$$

3.2. Perturbation Analysis. Using the method of multiple scales, τ is expanded in terms of $T_i = \varepsilon^i\tau$, $(i = 0, 1, \ldots)$. Therefore, the first and second time derivatives become

$$\begin{aligned}
\frac{d}{d\tau} &= D_0 + \varepsilon D_1 + \cdots, \\
\frac{d^2}{d\tau^2} &= D_0^2 + 2\varepsilon D_0 D_1 + \cdots,
\end{aligned} \tag{10}$$

where $D_i = \partial/\partial T_i$, $(i = 0, 1, \ldots)$.

The first-order approximate solutions of (8) take the following forms:

$$\begin{aligned}
\varphi_1(t,\varepsilon) &= \varphi_1^{(0)}\left(T_0, T_1\right) + \varepsilon\varphi_1^{(1)}\left(T_0, T_1\right), \\
x_c(t,\varepsilon) &= x_c^{(0)}\left(T_0, T_1\right) + \varepsilon x_c^{(1)}\left(T_0, T_1\right).
\end{aligned} \tag{11}$$

Substituting (10) and (11) into (8), then equating coefficients of like powers of ε, one obtains the following:

Order (ε^0):

$$\begin{aligned}
D_0{}^2\varphi_1^{(0)} + \varphi_1^{(0)} &= e_1 D_0{}^2\theta - e_8 D_0{}^2 x_c^{(0)}, \\
D_0{}^2 x_c^{(0)} + \omega_s{}^2 x_c^{(0)} &= -h_2 D_0{}^2\varphi_1^{(0)} - h_3 D_0{}^2\theta
\end{aligned} \tag{12}$$

Order (ε^1):

$$\begin{aligned}
&D_0{}^2\varphi_1^{(1)} + 2D_0 D_1\varphi_1^{(0)} + 2\xi_q D_0\varphi_1^{(0)} + \varphi_1^{(1)} \\
&\quad = e_2\left(D_0\theta\right)^2 + e_3\varphi_1^{(0)} D_0{}^2\varphi_1^{(0)} - e_4 x_c^{(0)} D_0{}^2\varphi_1^{(0)} \\
&\qquad + e_5\left(D_0\varphi_1^{(0)}\right)^2 - e_4 D_0 x_c^{(0)} D_0\varphi_1^{(0)} + e_7 D_0{}^2\theta\varphi_1^{(0)} \\
&\qquad - e_8 D_0{}^2 x_c^{(1)} - 2e_8 D_0 D_1 x_c^{(0)} - e_9 D_0{}^2\theta x_c^{(0)}, \\
&D_0{}^2 x_c^{(1)} + 2D_0 D_1 x_c^{(0)} + 2\omega_s\xi_c D_0 x_c^{(0)} + \omega_s{}^2 x_c^{(1)} \\
&\quad = h_1\left(D_0\varphi_1^{(0)}\right)^2 - h_2 D_0\varphi_1^{(1)} - 2h_2 D_0 D_1\varphi_1^{(0)} \\
&\qquad + h_4\left(D_0\theta\right)^2
\end{aligned} \tag{13}$$

The solution of (12) can be written in the form

$$\begin{aligned}
\varphi_1^{(0)} &= A_1\left(T_1\right)e^{j\omega_1 T_0} + A_2\left(T_1\right)e^{j\omega_2 T_0} + 0.5g_1 D_0{}^2\theta \\
&\quad + \overline{cc1}, \\
x_c^{(0)} &= \Gamma_1 A_1\left(T_1\right)e^{j\omega_1 T_0} + \Gamma_2 A_2\left(T_1\right)e^{j\omega_2 T_0} - \frac{h_3 D_0{}^2\theta}{\omega_S{}^2} \\
&\quad + \overline{cc2},
\end{aligned} \tag{14}$$

where $A_1(T_1)$ and $A_2(T_1)$ are functions of slow time T_1, $\overline{cc1}$ and $\overline{cc2}$ denote the complex conjugate terms, $\Gamma_1 = (1 - \omega_1{}^2)/(g_7\omega_1{}^2)$, and $\Gamma_2 = (1 - \omega_2{}^2)/(g_7\omega_2{}^2)$.

In this study, because the second-order nonlinear coupling terms exist in the dynamic model, the vibration absorber is used to control vibration of the flexible arm at the 2 : 1 internal resonance condition: that is, $\omega_2 \approx 2\omega_1$. In order to solve the nonlinear problem, the solutions of (13) take the following form:

$$\begin{aligned}
\varphi_1^{(1)} &= P_{11}e^{j\omega_1 T_0} + P_{12}e^{j\omega_2 T_0}, \\
x_c^{(1)} &= P_{21}e^{j\omega_1 T_0} + P_{22}e^{j\omega_2 T_0},
\end{aligned} \tag{15}$$

where P_{11}, P_{12}, P_{21}, and P_{22} are undetermined coefficients.

In the case of the 2 : 1 internal resonance, a detuning parameter σ is introduced; then

$$\begin{aligned}
2\omega_1 T_0 &= \omega_2 T_0 - \varepsilon\sigma T_0 = \omega_2 T_0 - \sigma T_1, \\
\left(\omega_2 - \omega_1\right)T_0 &= \omega_1 T_0 + \varepsilon\sigma T_0 = \omega_1 T_0 + \varepsilon T_1.
\end{aligned} \tag{16}$$

Substituting (14)–(16) into (13), then equating the coefficients of $e^{j\omega_1 T_0}$ and $e^{j\omega_2 T_0}$ on both sides, one obtains,

$$\left(1-\omega_n{}^2\right)P_{1n}-e_8\omega_n{}^2P_{2n}=R_{1n},$$
$$-h_2\omega_n{}^2P_{1n}+\left(\omega_s{}^2-\omega_n{}^2\right)P_{2n}=R_{1n} \quad (n=1,2), \tag{17}$$

where

$$\begin{aligned} R_{11} &= -2j\omega_1A_1'-j\omega_1 2\xi_qA_1-e_3\omega_1{}^2\overline{A_1}A_2e^{j\sigma T_1} \\ &\quad -e_1e_3\omega_1{}^2D_0{}^2\theta A_1-e_3\omega_2{}^2\overline{A_1}A_2e^{j\sigma T_1} \\ &\quad +e_4\omega_1{}^2\Gamma_2A_2\overline{A_1}e^{j\sigma T_1}+e_4\omega_2{}^2\overline{\Gamma_1}A_2\overline{A_1}e^{j\sigma T_1} \\ &\quad -\frac{e_4h_3\omega_1{}^2A_1D_0{}^2\theta}{\omega_s{}^2}+2e_5\omega_1\omega_2A_2\overline{A_1}e^{j\sigma T_1} \\ &\quad -e_4\omega_1\omega_2\overline{\Gamma_1}\,\overline{A_1}A_2e^{j\sigma T_1} \\ &\quad -e_4\omega_1\omega_2\Gamma_2\overline{A_1}A_2e^{j\sigma T_1}+e_7D_0{}^2\theta A_1 \\ &\quad -2e_8j\omega_1\Gamma_1A_1'-e_9D_0{}^2\theta\Gamma_1A_1, \\ R_{12} &= -2j\omega_2A_2'-2j\omega_2\xi_qA_2-e_3\omega_1{}^2A_1{}^2e^{-j\sigma T_1} \\ &\quad -e_3e_1D_0{}^2\theta\omega_2{}^2A_2+e_4\omega_1{}^2\Gamma_1A_1{}^2e^{-j\sigma T_1} \\ &\quad -\frac{e_4h_3\omega_2{}^2A_2D_0{}^2\theta}{\omega_s{}^2}-e_5\omega_1{}^2A_1{}^2e^{-j\sigma T_1} \\ &\quad +e_4\Gamma_1\omega_1{}^2A_1{}^2e^{-j\sigma T_1}+e_7D_0{}^2\theta A_2 \\ &\quad -2j\omega_2e_8\Gamma_2A_2'-e_9D_0{}^2\theta\Gamma_1A_1, \\ R_{21} &= -2j\omega_1\Gamma_1A_1'-2j\omega_1\omega_s\xi_c\Gamma_1A_1 \\ &\quad +2h_1\omega_1\omega_2A_2\overline{A_1}e^{j\sigma T_1}-2h_2j\omega_1A_1', \\ R_{22} &= -2j\omega_2\Gamma_2A_2'-2j\omega_2\omega_s\xi_c\Gamma_2A_2-2h_2j\omega_2A_2' \\ &\quad -h_1\omega_1{}^2A_1{}^2e^{-j\sigma T_1}. \end{aligned} \tag{18}$$

Therefore, the problem of determining the solvability conditions of (13) is reduced to that of determining the solvability condition of (17).

Since the determinant of the coefficient matrix of (17) is zero, the solvability conditions are

$$\begin{vmatrix} 1-\omega_n{}^2 & R_{1n} \\ -h_2\omega_n{}^2 & R_{2n} \end{vmatrix}=0 \quad \text{or } R_{1n}=\frac{\omega_n{}^2-1}{h_2\omega_n{}^2}R_{2n}. \tag{19}$$

It is convenient to express the resulting modulation equations in polar form by introducing the following transformation:

$$A_1=\frac{1}{2}a_1e^{j\theta_1},\qquad A_2=\frac{1}{2}a_2e^{j\theta_2}, \tag{20}$$

where a_1, a_2, θ_1, and θ_2 are real functions of the slow time T_1; a_1 and a_2 are defined as the modal amplitudes.

Substituting (20) into (18), then substituting (18) into (17), and setting the coefficients of the real and imaginary parts to zero yield the modulation equations:

$$a_1'=M_{11}a_1-0.5M_{13}a_1a_2\sin\gamma, \tag{21}$$

$$a_2'=M_{21}a_2+0.5M_{23}a_1{}^2\sin\gamma, \tag{22}$$

$$\gamma'=M_{22}+\frac{0.5M_{23}a_1{}^2}{a_2\cos\gamma}+\sigma-2M_{12}-M_{13}a_2\cos\gamma, \tag{23}$$

where

$$\gamma=\theta_2+\sigma T_1-2\theta_1,$$

$$M_{11}=\frac{-2\xi_q+\left(\omega_1{}^2-1\right)/\left(h_2\omega_1{}^2\right)2\xi_c\omega_s\Gamma_1}{2+2e_8\Gamma_1-(\omega_1{}^2-1)/(h_2\omega_1{}^2)2\Gamma_1-(\omega_1{}^2-1)/(h_2\omega_1{}^2)2h_2},$$

$$M_{12}=\frac{e_1e_3\omega_1{}^2D_0{}^2+e_4/\omega_s{}^2h_3\omega_1{}^2D_0{}^2\theta-e_7D_0{}^2\theta+e_9\Gamma_1D_0{}^2\theta}{\omega_1\left(2+2e_8\Gamma_1-(\omega_1{}^2-1)/(h_2\omega_1{}^2)2\Gamma_1-(\omega_1{}^2-1)/(h_2\omega_1{}^2)2h_2\right)},$$

$$M_{13}=\frac{e_3\omega_1{}^2+e_3\omega_2{}^2-e_4\omega_1{}^2\Gamma_2-e_4\omega_1{}^2\overline{\Gamma_1}-2e_5\omega_1\omega_2+e_4\omega_1\omega_2\overline{\Gamma_1}+e_4\omega_1\omega_2\Gamma_2+\left(\omega_1{}^2-1\right)/\left(h_2\omega_1{}^2\right)2h_1\omega_1\omega_2}{\omega_1\left(2+2e_8\Gamma_1-(\omega_1{}^2-1)/(h_2\omega_1{}^2)2\Gamma_1-(\omega_1{}^2-1)/(h_2\omega_1{}^2)2h_2\right)},$$

$$M_{21}=\frac{-2\xi_q+\left(\omega_2{}^2-1\right)/\left(h_2\omega_2{}^2\right)2\xi_c\omega_s\Gamma_2}{2+2e_8\Gamma_2-(\omega_2{}^2-1)/(h_2\omega_2{}^2)2\Gamma_2-(\omega_2{}^2-1)/(h_2\omega_2{}^2)2h_2},$$

$$M_{22} = \frac{e_1 e_3 D_0{}^2 \theta \omega_2{}^2 + e_4 h_3 D_0{}^2 \theta \omega_2{}^2 / \omega_s{}^2 - e_7 D_0{}^2 \theta + e_9 D_0{}^2 \theta \Gamma_2}{\omega_2 \left(2 + 2e_8 \Gamma_1 - (\omega_2{}^2 - 1) / (h_2 \omega_2{}^2)\, 2\Gamma_1 - (\omega_2{}^2 - 1) / (h_2 \omega_2{}^2)\, 2h_2\right)},$$

$$M_{23} = \frac{e_3 \omega_1{}^2 - e_4 \omega_1{}^2 \Gamma_1 + e_5 \omega_1{}^2 - e_4 \omega_1{}^2 \Gamma_1 - \left(\omega_2{}^2 - 1\right) / \left(h_2 \omega_2{}^2\right) h_1 \omega_2{}^2}{\omega_2 \left(2 + 2e_8 \Gamma_1 - (\omega_2{}^2 - 1) / (h_2 \omega_2{}^2)\, 2\Gamma_1 - (\omega_2{}^2 - 1) / (h_2 \omega_2{}^2)\, 2h_2\right)}. \tag{24}$$

4. Principle of Vibration Reduction

4.1. Establishment of Internal Resonance. In order to reduce vibration, it is important to establish the internal resonance. Under the internal resonance condition, the vibration energy can be transferred from the flexible arm to the vibration absorber. For this purpose, the undamped case (i.e., $\xi_q = 0$ and $\xi_c = 0$) is studied. Equations (21) and (22) are transferred into

$$a_1' = -0.5 M_{13} a_1 a_2 \sin\gamma, \tag{25}$$

$$a_2' = 0.5 M_{23} a_1{}^2 \sin\gamma. \tag{26}$$

Multiplying (25) by a_1 and (26) by $v a_2$ and then adding and integrating yield

$$a_1^2 + v a_2^2 = E, \tag{27}$$

where $v = M_{13}/M_{23}$ and E is a constant depending on initial conditions.

It can be seen that v is determined by the structural parameters of both the flexible arm and the vibration absorber. If the structural parameters of the flexible arm are given, then v will be uniquely determined by those of the vibration absorber, that is, m_0, l_0, and r_0. Therefore, it is easy to find the appropriate structural parameters to make $v > 0$.

If $v > 0$, it means that a_1 and a_2 in (27) are bounded and antiphase with each other. Since a_1^2 and a_2^2 indicate vibrational energy of the flexible arm and the vibration absorber, respectively, (27) indicates that, in the absence of the damping, the system is conservative. If a_1^2 decreases, then a_2^2 increases, and vice versa. This phenomenon has proven that the internal resonance has been established and the vibrational energy can be transferred between the flexible arm and the vibration absorber.

4.2. Vibration Absorption. After the internal resonance has been established, in order to reduce vibration of the flexible arm, the damping of the vibration absorber should be taken into account. In the presence of damping (i.e., $\xi_q > 0$ and $\xi_c > 0$), the steady-state response of (21) and (22) is

$$a_1' = a_2' = 0. \tag{28}$$

That is,

$$\begin{gathered} M_{11} a_1 - 0.5 M_{13} a_1 a_2 \sin\gamma = 0, \\ M_{21} a_2 + 0.5 M_{23} a_1{}^2 \sin\gamma = 0. \end{gathered} \tag{29}$$

It is easy to find that the system possesses the equilibrium points defined by

$$\begin{gathered} a_1 = 0, \\ a_2 = 0. \end{gathered} \tag{30}$$

Therefore, by examining the Jacobian, one can ascertain the stability of the system.

The Jacobian matrix of this case is

$$\begin{bmatrix} M_{11} & 0 & 0 \\ 0 & M_{21} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{31}$$

where M_{11} and M_{21} are the eigenvalues.

It can be seen that M_{11} and M_{21} are related to the structural parameters of both the flexible arm and the vibration absorber. If the structural parameters of the flexible arm are given, then they will be determined by those of the vibration absorber. Therefore, it is easy to find the appropriate structural parameters to make $M_{11} < 0$ and $M_{21} < 0$ within certain ranges. Due to the negative eigenvalues, the modal amplitudes a_1 and a_2 are stable. In this case, the vibrational energy of the flexible arm can be absorbed and dissipated by the vibration absorber.

It should be noted that this method is different from conventional semiactive methods. Firstly, those methods concern linear vibration problem, while this method can handle nonlinear vibration problem. Secondly, those methods pay close attention to a lumped flexible structure without rigid motion, while this method is competent for a flexible arm characterized by distributed flexibility and rigid motion. In addition, those methods linearly neutralize vibration of the primary system by means of their own vibration. But this method utilizes the internal resonance to establish a nonlinear modal coupling channel, thereby transferring vibration energy of the flexible arm to the MRE vibration absorber. Finally, those methods are implemented according to external excitations and thus have to rely on the information of external excitations. But this method aims to establish an internal channel for transferring vibration energy from the flexible arm to the MRE vibration absorber rather than responding directly to external excitations and thus is especially suitable for such case as the outer space where external excitations are unknown or unpredictable.

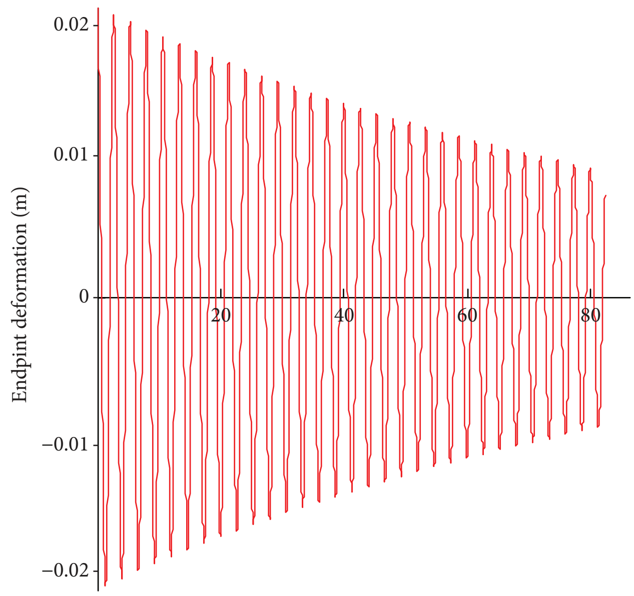

FIGURE 2: Vibration response of the uncontrolled arm.

5. Simulations and Analyses

5.1. Numerical Simulations. To verify the above theoretical analysis, some numerical simulations are conducted on the following conditions. The flexible arm: $l = 800$ mm, $h = 50$ mm, $b = 3$ mm, $\rho = 7900$ kg/m^3, and $E = 2.1 \times 10^{11}$ Pa. The vibration absorber: $m_0 = 1.24$ kg, $l_0 = 90$ mm, and $r_0 = 230$ mm.

Suppose the desired joint motion of the manipulator is

$$\theta = \frac{\pi}{3}\sin(0.01\pi t). \tag{32}$$

If the flexible arm is not equipped with the vibration absorber, given the initial disturbance of 20 mm, its endpoint response is shown in Figure 2 when moving according to (32). Due to small damping in the flexible arm, its endpoint response attenuates very slowly.

To control nonlinear vibration, the flexible arm is equipped with a vibration absorber, whose frequency is tuned via the MRE to be twice as much as the flexible arm's fundamental frequency. If the damping of the vibration absorber is not taken into account, the modal amplitudes a_1 and a_2 are obtained by numerically integrating (21), (22), and (23). As shown in Figure 3, a_1 (dashed line) and a_2 (solid line) are exactly antiphased. It means that, when a_1 decreases to the minimum, a_2 increase to the maximum at the same time, and vice versa. Since the vibration energy is related to the amplitude, the above phenomenon indicates that the vibration energy is transferring between the flexible arm and the vibration absorber. It is proven that the internal resonance has been successfully established between the fundamental mode of the flexible arm and the mode of the vibration absorber.

Then the damping of the vibration absorber is taken into account: for example, $\xi_c = 0.05$; the modal amplitudes a_1 and a_2 are obtained by numerically integrating (21), (22), and (23). As shown in Figure 4, the modal amplitude of the flexible arm (dashed line) is reduced rapidly. It means that the vibration energy of the flexible arm is dissipated effectively. From Figure 5, it can be seen that the endpoint vibration

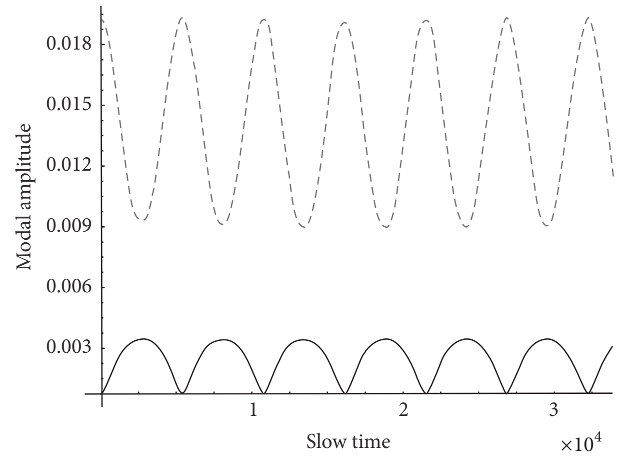

FIGURE 3: Undamped modal amplitudes.

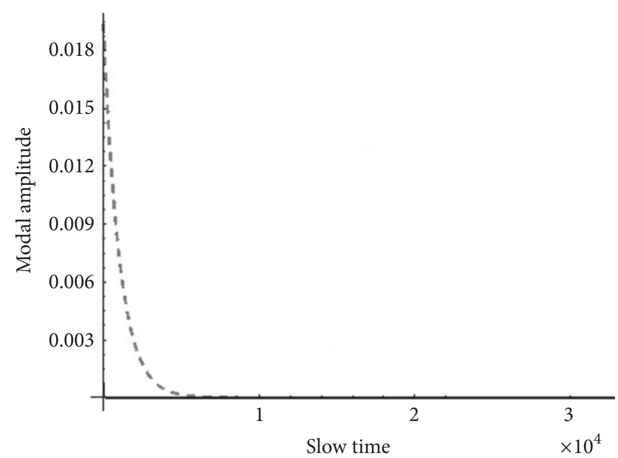

FIGURE 4: Undamped modal amplitudes.

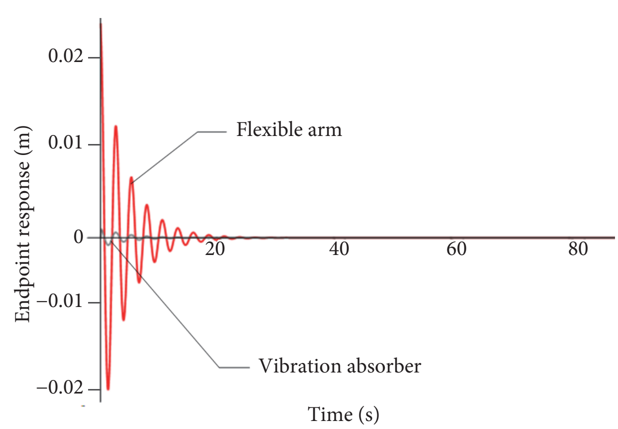

FIGURE 5: Endpoint response of the flexible arm.

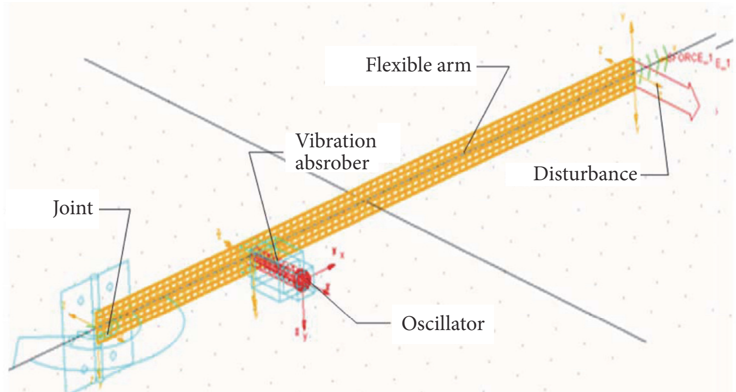

Figure 6: Dynamics model of the system established in ADAMS.

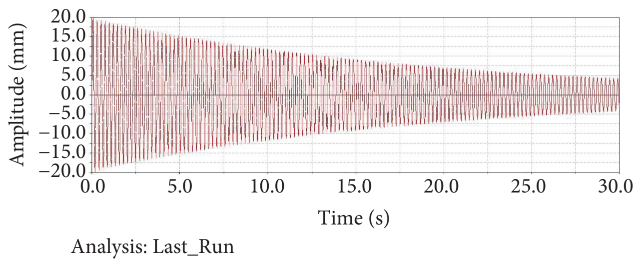

Figure 7: Vibration response of the flexible arm (no vibration absorber).

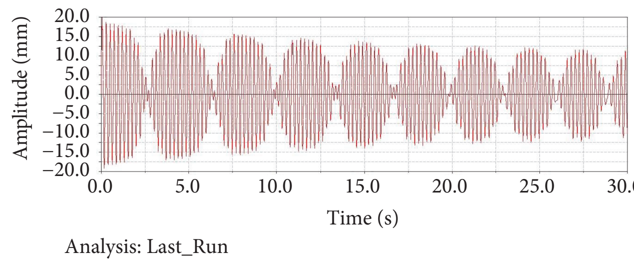

Figure 8: Vibration response of the flexible arm (no damping).

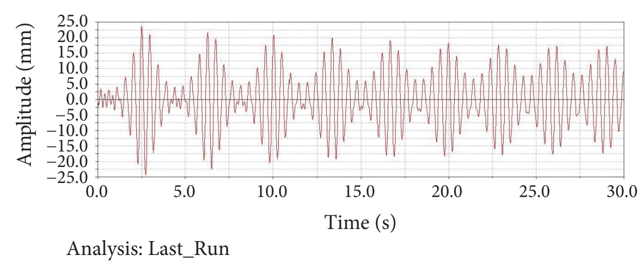

Figure 9: Vibration response of the vibration absorber (no damping).

response of the flexible arm has been successfully attenuated. After 10 seconds, the deformation has been eliminated by 80%. Moreover, there is no residual vibration left.

5.2. Virtual Prototyping Simulations. Although the above numerical simulations have achieved satisfactory results, they are based on our own theoretical model. To further verify theoretical analysis, several virtual prototyping simulations are conducted using ADAMS software (Automatic Dynamic Analysis of Mechanical System, MSC Software Corp.). Dynamics modeling and solution for the flexible arm and the vibration absorber are implemented by ADAMS software, from which more authentic and convincing results can be obtained.

Dynamics models of the flexible arm and the vibration absorber are established using ANSYS software, as shown in Figure 6. Suppose the rigid motion of the flexible arm is the same as the examples in theoretical simulations. Given the initial disturbance 20 mm, if the flexible arm is not equipped with the vibration absorber, the endpoint response attenuates very slowly due to small damping, as shown in Figure 7.

To control the vibration of the flexible arm, a vibration absorber is attached to the flexible arm. In order to verify whether the internal resonance can be established, the damping of the vibration absorber is not taken into account. In this case, the vibration responses of the flexible arm and the vibration absorber are obtained under the 2 : 1 internal resonance condition and shown in Figures 8 and 9, respectively. It is seen that when the amplitude of the flexible arm is decreasing, the amplitude of the vibration absorber is increasing, and vice versa. Furthermore, if the former decreases to the minimum, then the latter increases to the maximum, and vice versa. It means that the vibration energy is exchanging between the fundamental mode of the flexible arm and the mode of the vibration absorber.

If the damping of the vibration absorber is taken into account, for example, 0.066 in ADAMS, the vibration response of the flexible arm is shown in Figure 10. It takes only about 5 seconds to decrease 90% of the initial vibration amplitude. Compared with Figure 7, it can be seen that the vibration response of the flexible arm can be effectively reduced with the help of the damping of the vibration absorber.

Furthermore, if the frequency ratio between the vibration absorber and the flexible arm slightly deviates the 2 : 1 internal

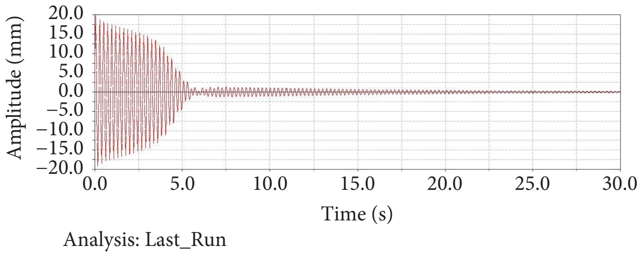

Figure 10: Vibration response of the flexible arm subjected to the damping of the vibration absorber.

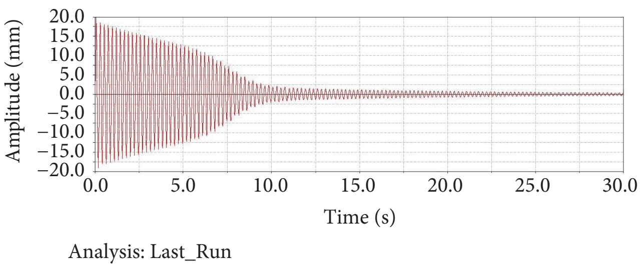

Figure 11: Vibration response of the flexible arm under the frequency ratio of 2.2 : 1.

resonance condition, for example, being 2.2 : 1, the vibration response of the flexible arm is shown in Figure 11. As can be seen, its response is larger than that in the 2 : 1 internal resonance condition (as shown in Figure 10) due to the deviation of the frequency ratio. It demonstrates the effectiveness of the 2 : 1 internal resonance. Even so, its response is still much better than that in the uncontrolled case, as compared with Figure 7. It indicates that this vibration absorber possesses certain robustness.

Based on the above virtual prototyping simulations, it is verified that this method is able to achieve satisfactory results in reducing nonlinear vibration of the flexible arm.

6. Development of MRE Vibration Absorber

Based on the above theoretical study, a MRE vibration absorber is developed. The MRE element serves as a smart spring whose frequency can be tuned to satisfy the internal resonance condition with the flexible arm, thereby attenuating nonlinear vibration of the flexible arm.

6.1. MRE Material Preparation. The MRE material used in this study is composed of the carbonyl iron particles (3–5 μm, 75% mass ratio) provided by Beijing Xingrongyuan Ltd., the 704 silicone rubber provided by Changzhou Nanda Ltd., and a small amount of silicone oil provided by Hangzhou West Lake organic silicon factory. The fabricating devices include an agitator (Figure 12), a static magnetic field device of 1T (Figure 13), a vacuum deaeration tank (Figure 14), and an aluminum alloy mold (Figure 15).

The fabricating process is as follows. Firstly, the carbonyl iron particles, the 704 silicone rubber, and a small amount of silicone oil are sufficiently blended using an agitator. Then

Figure 12: The agitator.

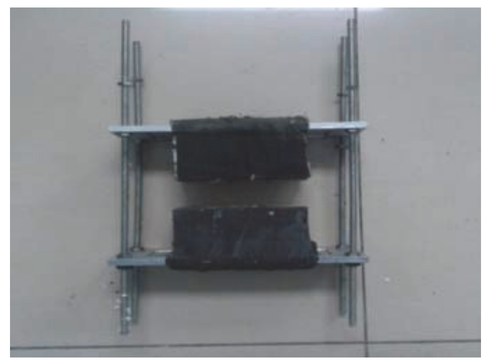

Figure 13: Static magnetic field device.

Figure 14: Vacuum deaeration tank.

Figure 15: Aluminum alloy mold.

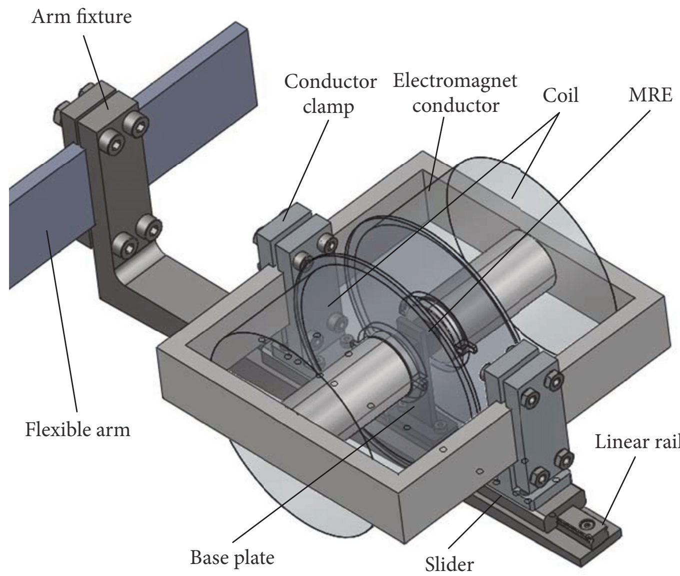

FIGURE 16: Scheme of the MRE vibration absorber.

the mixture is placed in the vacuum deaeration tank to remove the air in the mixture. Afterwards, it is packed into an aluminum alloy mold and placed in the magnetic field of 1T for 24 hours. Via the magnetic forces, the iron particles will be arranged in chain and exhibit good MR effects.

6.2. MRE Vibration Absorber. As shown in Figure 16, the vibration absorber in this study consists of the enclosed double E-shape electromagnet conductor, the coils, the linear rail, the slider, the MRE elements, the base plate, the arm fixture, the conductor clamp, and so on. The electromagnet conductor and two coils are used to create the magnetic field, whose strength is controlled by the coil current provided by an external DC power. The base plate is fixed on the linear rail mounted on the arm fixture. The one side of the MRE element is stuck to the base plate, and the other side is stuck to the electromagnet conductor. The electromagnet conductor and the coils are used as an oscillator, which are installed on the slider and can reciprocally move along the linear rail. Therefore, the MRE elements can work in the shear mode. Since the MRE's shear modulus depends on the magnetic field strength, it can serve as a smart spring element. As a result, the frequency of the vibration absorber based on the MRE can be readily controlled by the coil current.

7. Experimental Study

In this section, several experimental studies are conducted using the aforementioned MRE vibration absorber to investigate the proposed method.

7.1. Experimental Setup. As shown in Figures 17 and 18, an experimental setup is designed and composed of a flexible arm, a MRE vibration absorber, and a vibration analysis system. The flexible arm possesses the same structural parameters as the numerical simulation model and is driven by a servomotor controlled by the PMAC® motion controller and the supervisory computer. The MRE vibration absorber is installed on the flexible arm. A controllable DC power is used to adjust the frequency of the MRE vibration absorber. The vibration analysis system consists of the accelerometer and the dynamic signal analyzer. The B&K® accelerometer is stuck to the end of the flexible arm and used to collect the acceleration signals. The dynamic signal analyzer is used to process the acceleration signals and then transfer them to the supervisory computer.

7.2. Experimental Investigation. If the 20 mm initial disturbance is exerted on the flexible arm when moving according to (32), the acceleration signals of the endpoint are detected by the accelerometer and processed by the dynamic signal analyzer, as shown in Figure 19. In the absence of the vibration absorber, the endpoint response attenuates very slowly. It takes about 11 s to decrease 80% of the initial amplitude.

In order to reduce vibration of the flexible arm, the 2 : 1 internal resonance should be established. To this end, the frequency of the MRE vibration absorber is adjusted through the coil current to be 10.7 Hz which is twice as much as the flexible arm's fundamental frequency, as shown in Figure 20.

If the flexible arm is equipped with the MRE vibration absorber, when subjected to the same initial disturbance, nonlinear vibration of the flexible arm can be effectively reduced, as shown in Figure 21. It takes only 4 s to decrease 80% of the initial vibration amplitude. Compared with Figure 19, the efficiency of vibration reduction has increased by 64%. Moreover, there is no residual vibration left.

If the frequency of the MRE vibration absorber is tuned to be 9.63 Hz through adjusting its current, that is, the frequency ratio between the vibration absorber and the flexible arm is

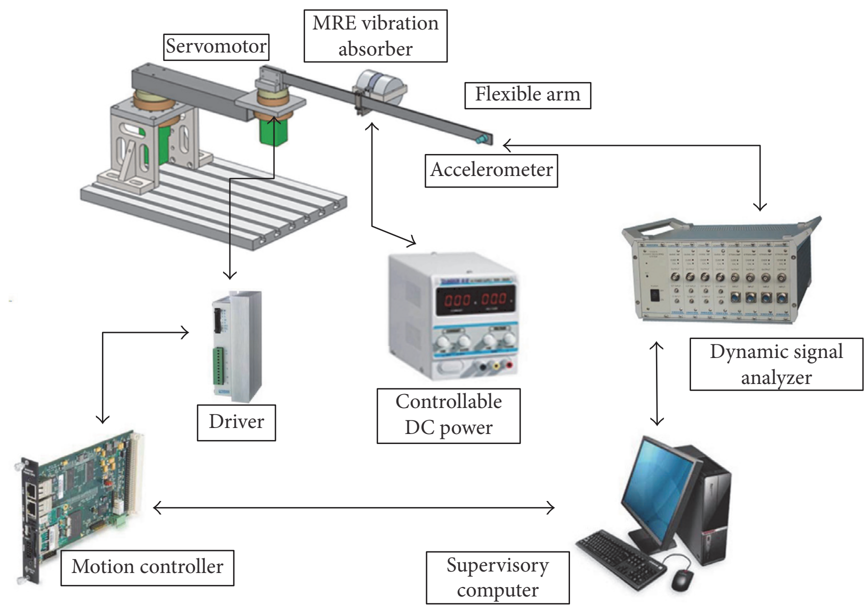

Figure 17: Scheme of the experimental setup.

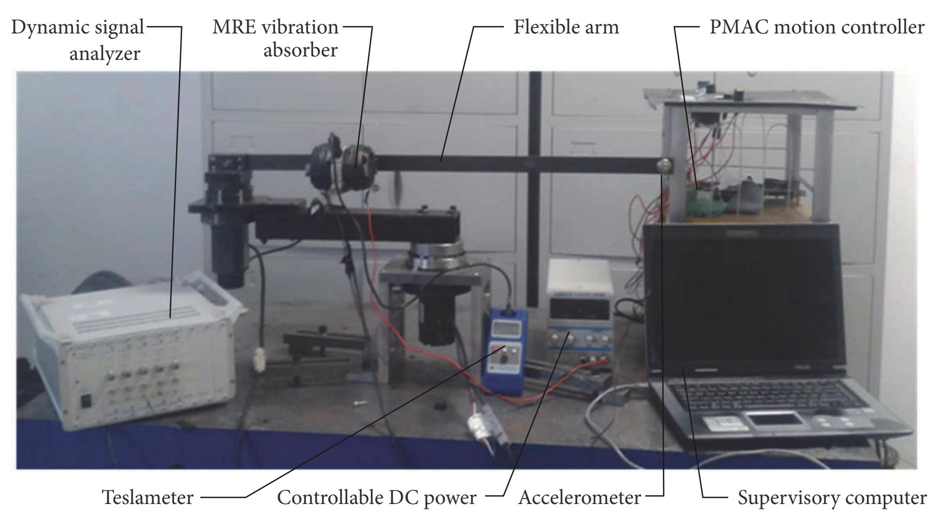

Figure 18: Photograph of the experimental setup.

slightly smaller than 2 : 1 internal resonance condition, the vibration response of the flexible arm is shown in Figure 22. As can be seen, its response is larger than that in the 2 : 1 internal resonance condition (as shown in Figure 21) due to the deviation of the frequency ratio.

On the other hand, if the frequency of the MRE vibration absorber is tuned to be 11.77 Hz through adjusting its current, that is, the frequency ratio between the vibration absorber and the flexible arm is slightly larger than 2 : 1 internal resonance condition, the vibration response of the flexible arm is shown in Figure 23. Similarly, its response is larger than that in 2 : 1 internal resonance condition (as shown in Figure 21) due to the deviation of the frequency ratio.

From Figures 22 and 23, it demonstrates the effectiveness of the 2 : 1 internal resonance. If the 2 : 1 internal resonance condition cannot be satisfied, vibration control performance will degrade. Even so, it is found that these responses are still much better than that in the uncontrolled case, as compared with Figure 19. Although the frequency of the vibration absorber alters ±10%, 80% of the initial vibration amplitude can still be effectively reduced within 6 s, and the efficiency of vibration reduction has increased about 45%. It indicates the robustness of this vibration absorber. Therefore, if the above efficiency of vibration reduction is acceptable, the working frequency width of the proposed vibration absorber is 2.14 Hz in this example.

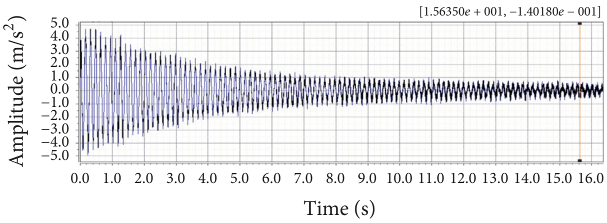

Figure 19: Response of the flexible arm without vibration absorber.

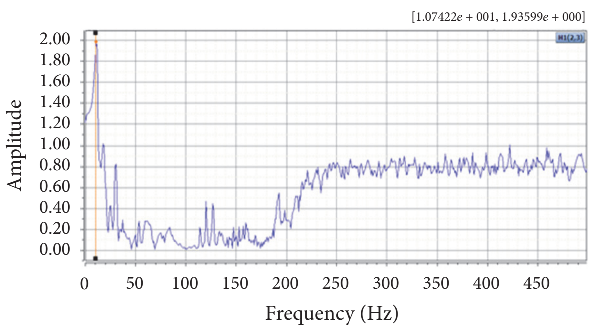

Figure 20: Frequency response of the MRE vibration absorber.

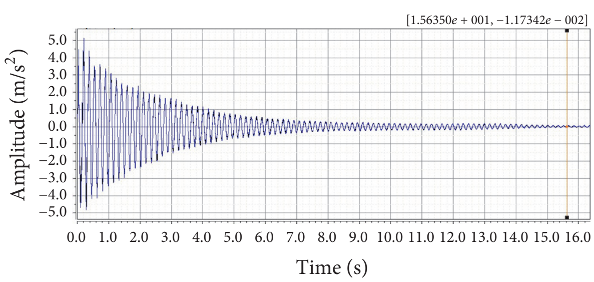

Figure 21: Response of the flexible arm with vibration absorber.

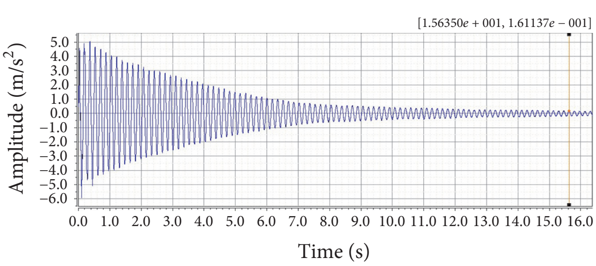

Figure 22: Response of the flexible arm under the smaller frequency ratio.

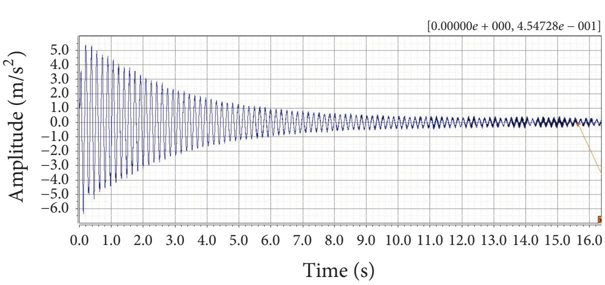

Figure 23: Response of the flexible arm under the larger frequency ratio.

From the above experimental results, it is verified that the proposed method and the MRE vibration absorber are effective and feasible to decrease vibration of the flexible arm.

8. Conclusion

A semiactive control method is put forward to attenuate nonlinear vibration of the flexible arm via the internal resonance and the MRE. A shear mode vibration absorber with MRE is developed to establish and maintain a vibration energy transfer channel and composed of an oscillator, smart spring elements (fabricated with MRE), an enclosed double E-shape electromagnet conductor, two coils, and so on. Its frequency can be readily controlled by adjusting the coil current. Under the 2 : 1 internal resonance condition, it is proven that the internal resonance can be successfully established between the fundamental mode of the flexible arm and the mode of the MRE vibration absorber. The vibration energy of the flexible arm can be transferred to and dissipated by the MRE vibration absorber via the modal interaction. It is verified by the simulations and experiments that the proposed method and the suggested MRE vibration absorber are effective in controlling nonlinear vibration of the flexible arm.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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